

DERIVED ANALYTIC GEOMETRY READING GROUP

DATE: Monday Weeks 1 - 8 · TIME: 2 - 3 PM · LOCATION: L4

OVERVIEW

Toën and Vezzosi explained in [7] that any symmetric monoidal category gives rise to an algebraic geometry relative to this category. In this reading group, we study several instances of such algebraic geometries relative to functional analytic categories, most notably the quasi-abelian categories of Banach-, Ind-Banach-, and Bornological spaces. This point of view recovers and unifies classical notions from analytic geometry, and opens the door to a derived analytic geometry, both Archimedean and non-Archimedean. These observations are due to Bambozzi, Ben-Bassat, Kelly, and Kremnitzer, whose work we will discuss in this reading group.

SCHEDULE

- TALK 1: The categories of Banach-, Ind-Banach-, and Bornological spaces (Most of the articles that we will study have a complete overview of these categories; see also [5])
- TALK 2: Introduction to relative algebraic geometry (See [8])
- TALK 3: Localisations of Affinoid algebras (See [1, Section 5.1] and [2, Section 4.1])
- TALK 4: Localisations of general Banach rings (See [1, Section 5.2] and [2, Section 4.2])
- TALK 5: Topologies in the Banach algebraic geometry setting (See [1, Section 5.3 and 5.4])
- TALK 6: The homotopical Huber spectrum of a Banach ring and localisations of bornological rings (See [2, Section 4.3,4.4, and 5])
- TALK 7: A perspective on the foundations of derived analytic geometry (See [3](preprint release TBC)).

TBC: A talk by Dr Jack Kelly of Trinity College Dublin on motivation for this theory.

Further references. We refer to [4] for an introduction to symmetric monoidal categories. The examples of such categories that we are interested in are usually not abelian, but *quasi-abelian*. This notion was introduced by Schneiders in [6].

REFERENCES

- [1] O. Ben-Bassat and K. Kremnitzer. Non-archimedean analytic geometry as relative algebraic geometry. *Annales de la faculté des sciences de Toulouse Mathématiques*, XXVI(1):49–126, 2017.
- [2] F. Bambozzi K. Kremnitzer. On the sheafyness property of spectra of Banach rings. *Arxiv e-prints [2009,13926v2]*, 2020.
- [3] O. Ben-Bassat J. Kelly K. Kremnitzer. A perspective on the foundations of derived analytic geometry. work in progress.
- [4] S. MacLane. *Categories for the Working Mathematician*, volume 5 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2 edition, 1998.
- [5] R. Meyer. *Local and Analytic Cyclic Homology*, volume 3 of *EMS Tracts in Mathematics*. European Mathematical Society, Switzerland, 2007.

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- [6] J-P. Schneiders. Quasi-abelian categories and sheaves. *Mémoires de la Société Mathématique de France*, 76, 1999.
- [7] B. Toen and M. Vaquie. Under spec \mathbb{Z} . *Journal of K-Theory*, 3:437 – 500, 2009.
- [8] B. Toen G. Vezzosi. Homotopical algebraic geometry ii; geometric stacks and applications. *Memoirs of the American Mathematical Society*, 193(902).