RAPOPORT-ZINK SPACES STUDY GROUP

DATE: Thursday Weeks 1 - 8 · TIME: 11:00-12:30 · LOCATION: C4

OVERVIEW

This study group is about Moduli Spaces of p-Divisible Groups. This term, we focus on the basic theory of Rapoport-Zink spaces. Next term, we will aim to understand the duality isomorphism of Rapoport-Zink spaces of EL type at infinite level, and how Rapoport-Zink spaces are generalised by local Shimura varieties.

SCHEDULE

The references we will most closely follow are [8], [12], following a similar plan to [5].

WEEK 1: **p-Divisible Groups**
Definition of p-divisible groups over a scheme and a formal scheme. Isogenies and quasi-isogenies. Mention relationship between p-divisible groups and formal groups. Dieudonné classification over a perfect field of characteristic p.
References: [12] §1.1 until Theorem 1.3.8, which itself follows [7]. [3] for Diéudonne classification, Thm. 7.2.4, Thm. 7.2.6.

WEEK 2: **Grothendieck-Messing Theory**
The crystal of a p-divisible group, universal vector extension, and the main theorem of Grothendieck-Messing Theory.
References: [12] discussion after Thm. 1.3.8 until end of §1.4, [7], [2] Thm. 12.1.5.

WEEK 3: **The Rapoport-Zink Moduli Problem and The Grothendieck-Messing Period Morphism**
Define the Moduli Problems of type (EL) and (PEL). Explain how the representability result Theorem 3.25 [8] is reduced to the universal case Theorem 2.16 [8].
References: Chapter 3 of [8], Chapter 2 of [12].

WEEK 4: **Rigid Generic Fibre, The Tower, and the Grothendieck-Messing Period Morphism**
Explain how the associated tower of rigid spaces is defined, and the group actions on the tower. Define the period morphism.
References: Chapter 5 of [8], Chapter 2 of [12], [10] Prop. 4.22 and Prop. 4.23.

WEEK 5: No Seminar

WEEK 6: **The Period Domain and The Image of the Grothendieck-Messing Period Morphism**
Define the period domain $\mathcal{F}(G, \{\mu\})$, associated to the pair $(G, \{\mu\})$. Define the weakly admissible admissible open subset. For a Rapoport Zink datum, state theorem regarding factorisation of the period morphism through the weakly admissible locus. See also [11], discussion on page 5.
References: [12] Def. 3.2.9, Thm. 3.2.11, [4], [10] Prop. 4.23, [8].

Date: January 9, 2023.

WEEK 7: **The Lubin-Tate Case**
Specifics for the Lubin-Tate case.
References: [1], [6], [8] section 3.78.

WEEK 8: **The Drinfeld Case**
Specifics for the Drinfeld case.
References: [2], [8] section 3.54 - 3.77, [9] for properties which are particular to the Drinfeld case.

REFERENCES

[1] Johannes Anschütz. *Lecture Notes:* The Lubin-Tate Tower.
[2] J.-F. Boutot and H. Carayol. Uniformisation p-adique des courbes de Shimura: les theoremes de cerednik et de Drinfeld. Number 196-197, pages 7, 45–158 (1992). 1991. Courbes modulaires et courbes de Shimura (Orsay, 1987/1988).
[3] Oliver Brinon and Brian Conrad. *Notes:* p-adic Hodge Theory.
[4] Jean-François Dat, Sascha Orlik, and Michael Rapoport. *Period domains over finite and p-adic fields*, volume 183 of *Cambridge Tracts in Mathematics*. Cambridge University Press, Cambridge, 2010.
[5] Ulrich Görtz. Seminar plan for seminar on Rapoport-Zink spaces. Available Online.
[6] M. J. Hopkins and B. H. Gross. Equivariant vector bundles on the Lubin-Tate moduli space. In *Topology and representation theory (Evanston, IL, 1992)*, volume 158 of *Contemp. Math.*, pages 23–88. Amer. Math. Soc., Providence, RI, 1994.
[7] William Messing. *The crystals associated to Barsotti-Tate groups: with applications to abelian schemes*. Lecture Notes in Mathematics, Vol. 264. Springer-Verlag, Berlin-New York, 1972.
[8] M. Rapoport and Th. Zink. *Period spaces for p-divisible groups*, volume 141 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 1996.
[9] Michael Rapoport. Non-Archimedean period domains. In *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Zürich, 1994)*, pages 423–434. Birkhäuser, Basel, 1995.
[10] Michael Rapoport and Eva Viehmann. Towards a theory of local Shimura varieties. *Münster J. Math.*, 7(1):273–326, 2014.
[11] Peter Scholze and Jared Weinstein. Moduli of p-divisible groups. *Camb. J. Math.*, 1(2):145–237, 2013.
[12] Haoran Wang. Thesis: Moduli spaces of p-divisible groups and period morphisms.